8.1 practice problem solutions

1 $8x^2 = 27y^3$ from $(1, \frac{2}{3})$ to $(8, \frac{8}{3})$

either solve for x or solve for y

$$27y^3 = 8x^2$$
$$y^3 = \frac{8x^2}{27}$$
$$y = \left(\frac{8}{27}\right)^{1/3}(x^2)^{1/3}$$
$$\boxed{y = \left(\frac{2}{3}\right)x^{2/3}}$$

$$y' = \frac{2}{3}\left(\frac{2}{3}x^{-1/3}\right)$$
$$\boxed{y' = \frac{4}{9}x^{-1/3}}$$

Let's clean it up first

$$\sqrt{1+(y')^2} = \sqrt{1+\left(\frac{4}{9}x^{-1/3}\right)^2} = \sqrt{1+\frac{16}{81}x^{-2/3}}$$

$$= \sqrt{1+\frac{16}{81x^{2/3}}} = \sqrt{\frac{81x^{2/3}+16}{81x^{2/3}}} = \frac{1}{9x^{1/3}}\sqrt{81x^{2/3}+16}$$

$$L = \int_1^8 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_1^8 \frac{1}{9}x^{-\frac{1}{3}}\left(81x^{2/3}+16\right)^{1/2}dx$$

solve using u-substitution

$$= \boxed{\left(4+\frac{16}{81}\right)^{3/2} - \left(1+\frac{16}{81}\right)^{3/2}}$$

2 $y = 5 - \sqrt{x^3} = 5 - x^{3/2}$ $(1,4)$ to $(4,-3)$

$y' = -\frac{3}{2}x^{1/2}$

let's clean up $\sqrt{}$ first

$$\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+\left(-\frac{3}{2}x^{1/2}\right)^2}$$

$$= \sqrt{1+\frac{9}{4}x}$$

$$L = \int_1^4 \sqrt{1+\frac{9}{4}x}\,dx$$ use u substitution.

$$L = \frac{4}{9}\int_1^4 \left(1+\frac{9}{4}x\right)^{1/2} \frac{9}{4}dx = \frac{4}{9}\left[\frac{2}{3}\left(1+\frac{9x}{4}\right)^{3/2}\right]_1^4$$

$$= \frac{8}{27}\left[\left(1+\frac{9}{4}(4)\right)^{3/2} - \left(1+\frac{9}{4}(1)\right)^{3/2}\right]$$

$$= \boxed{\frac{8}{27}\left[10^{3/2} - \left(\frac{13}{4}\right)^{3/2}\right]}$$

3 $x = \frac{1}{16}y^4 + \frac{1}{2}y^{-2}$ $\left(\frac{9}{8}, -2\right)$ to $\left(\frac{9}{16}, -1\right)$

$$\frac{dx}{dy} = \frac{1}{16}(4)y^3 + \frac{1}{2}(-2)y^{-3}$$

$$\frac{dx}{dy} = \frac{1}{4}y^3 - y^{-3} = \frac{y^3}{4} - \frac{1}{y^3} = \frac{y^6-4}{4y^3}$$

lcd $= 4y^3$

prep work

$$\sqrt{1+\left(\frac{dx}{dy}\right)^2} = \sqrt{1+\left(\frac{y^6-4}{4y^3}\right)^2}$$

$$= \sqrt{1+\frac{(y^6-4)^2}{16y^6}} = \sqrt{\frac{16y^6+(y^{12}-8y^6+16)}{16y^6}}$$

$$= \frac{1}{4y^3}\sqrt{y^{12}+8y^6+16} = \frac{1}{4y^3}\sqrt{(y^6+4)^2}$$

$$= \frac{y^6+4}{4y^3} = \frac{1}{4}y^3 + \frac{1}{y^3} = \boxed{\frac{1}{4}y^3 + y^{-3}}$$

so

$$L = \int_{-2}^{-1}\left(\frac{1}{4}y^3+y^{-3}\right)dy = \frac{1}{4}\frac{y^4}{4} + \frac{y^{-2}}{-2}\Bigg|_{-2}^{-1}$$

$$= \frac{1}{16}y^4 - \frac{1}{2y^2}\Bigg|_{-2}^{-1}$$

$$= \left(\frac{1}{16}(-1)^4 - \frac{1}{2(-1)^2}\right) - \left(\frac{1}{16}(-2)^4 - \frac{1}{2(-2)^2}\right)$$

$$\frac{1}{16} - \frac{1}{2} - \left(1 - \frac{1}{8}\right)$$

$$\frac{1}{16} - \frac{8}{16} - \frac{16}{16} + \frac{2}{16}$$

$$\frac{21}{16}$$

4 Set up only

$2y^3 - 7y + 2x - 8 = 0 \qquad (3,2) \text{ to } (4,0)$

solve for x $\quad 2x = -2y^3 + 7y + 8$

$$\boxed{x = -y^3 + \frac{7}{2}y + 4}$$

$$\boxed{\frac{dx}{dy} = -3y^2 + \frac{7}{2}}$$

the prep work

$$\sqrt{1 + \left(\frac{dx}{dy}\right)^2} = \sqrt{1 + \left(\frac{7}{2} - 3y^2\right)^2}$$

$$= \sqrt{1 + \left(\frac{49}{4} - \frac{42}{2}y^2 + 9y^4\right)}$$

$$= \sqrt{\frac{4}{4} + \frac{49}{4} - \frac{84}{4}y^2 + \frac{36y^4}{4}}$$

$$= \frac{1}{2}\sqrt{53 - 84y^2 + 36y^4}$$

$$\boxed{L = \frac{1}{2}\int_2^0 \sqrt{36y^4 - 84y^2 + 53}\, dy}$$

or

$$\boxed{L = \int_0^2 \sqrt{9y^4 - 21y^2 + \frac{53}{4}}\, dy}$$

$\boxed{\text{Set up only}}$

5 $\quad 11x - 4x^3 - 7y + 7 = 0$ from $(1,2)$ to $(0,1)$

Solve for y

$$-7y = -11x + 4x^3 - 7 \qquad \boxed{y = 1 + \frac{11}{7}x - \frac{4}{7}x^3}$$

$$y = \frac{11}{7}x - \frac{4}{7}x^3 + 1 \qquad y' = \frac{11}{7} - \frac{4}{7}(3)x^2$$

$$\boxed{y' = \frac{11}{7} - \frac{12}{7}x^2}$$

$\boxed{\text{the prep work}}$

$$\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1 + \left(\frac{11}{7} - \frac{12}{7}x^2\right)^2}$$

$$= \sqrt{1 + \left(\frac{121}{49} - \frac{264x^2}{49} + \frac{144}{49}x^4\right)}$$

$$= \sqrt{\frac{49}{49} + \frac{121}{49} - \frac{264x^2}{49} + \frac{144x^4}{49}}$$

$$= \frac{1}{7}\sqrt{170 - 264x^2 + 144x^4}$$

$$L = \frac{1}{7}\int_0^1 \sqrt{170 - 1848x^2 + 144x^4}\, dx$$

It is OK to use lowest to highest x despite the discrepency in order.

you do not need to use the property $\int_a^b = -\int_b^a$ in this case.